

Abstract

A mathematical study about the sustaining phenomenon of overtone in flageolet harmonics on bowed string instruments

Shodai Tanaka (Sapporo Kaisei Secondary School Grade 5)

Flageolet harmonics is a playing technique on bowed string instruments, in which a player lightly touches a nodal point on a string with their finger. Previous studies have reported that the harmonic sound sustains for a short time after the finger is removed from the string during flageolet harmonics was performed on bowed string instruments. However, the mechanism of this harmonics-sustaining phenomenon and the parameter dependency of its sustaining time remain unclear. The purpose of this study is to mathematically investigate the dependence of sustaining time on parameters, and thereby, to elucidate the mechanism of the harmonics-sustaining phenomenon.

To this end, a mathematical model was devised by adding terms representing the effects of a bow and a finger to the one-dimensional wave equation. Subsequently, numerical simulation was performed to analyze the behavior of the model.

The devised model successfully reproduced the harmonics-sustaining phenomenon in which the parameter dependence of sustaining time was qualitatively consistent with the author's empirical observations. It was found that the parameter dependence of sustaining time follows the power law. Furthermore, dimensional analysis was performed, yielding the proposing of a formula which expresses the relationship between the sustaining time and the maximum and minimum bow force required to generate Helmholtz motion. As the result, it was predicted that the essence of the parameter dependency of sustaining time is "how the bow obstructs the string behavior".

This study provides a scientific explanation of the harmonics-sustaining phenomenon for the benefit of all players and lovers of bowed string instruments.